EIROPAS SOCIĀLAIS FONDS

EIROPAS SAVIENĪBA

IEGULDĪJUMS TAVĀ NĀKOTNĒ

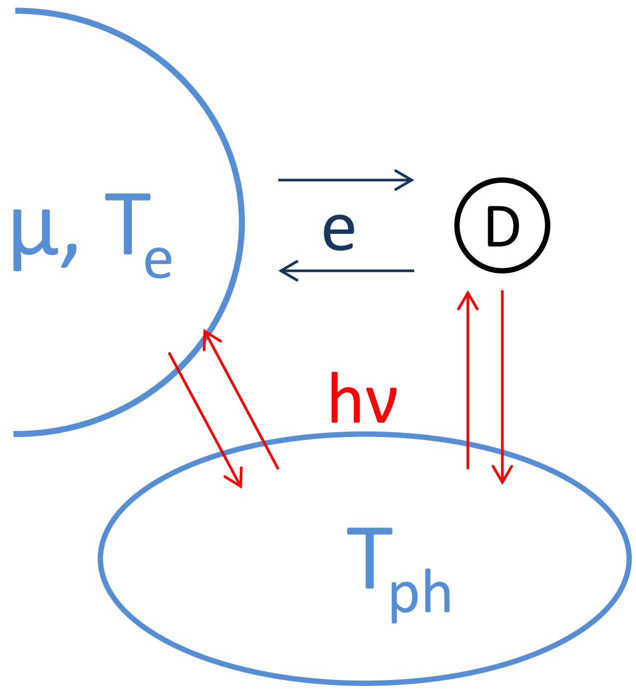
Lādiņa pārnese kvantu sūkņos: kvantu punkta temperatūras ietekme

Pāvels Nazarovs

Eiropas Savienības struktūrfondu projekta
Nr. 2009/0216/1DP/1.1.1.2.0/09/APIA/VIAA/044
„Datorzinātnes pielietojumi un tās saiknes ar kvantu fiziku” ietvaros

21/12/2011

Small systems: Statistical ensembles



<u>Equilibrium</u>

- microcanonical ensemble
- defined P_n and P(E)

<u>Non-equillibrium</u>

- unknown P_n(E, t)
- solvable, but difficult

<u>Fast coupling to thermal bath</u>

- $T_{dot} = T_{ph} = T_e$
- fast relaxation in the dot
- canonical ensemble
- $P_n(E,t) = P_n(t)P_{canon}(E,n)$
- unkown P_n(t)
- solvable and easier

<u>Main assumptions:</u>

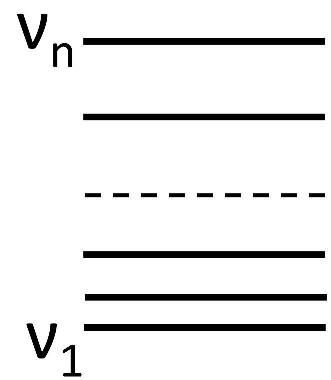
- electron exchange with the lead (P_n(t))
- energy relaxation in the dot is much faster than typical time between tunneling events (equilibrium probability distribution in the dot for fixed n)
- energy exchange with thermal bath ($T_{everywhere}$ = const, canonical ensemble)

Small systems: Kinetic equation

$$\frac{dP_n}{dt} = P_{n+1}W_{n+1}^{-} + P_{n-1}W_{n-1}^{+} - P_nW_n^{-} - P_nW_n^{+}$$

Small systems: Structure of the dot

$\nu_i = 0$ or 1



Constant interaction model

- electron adds only charging energy E_C
- single-particle energy spectrum is not affected by any other interactions inside the dot
- n independently occupied levels out of many

$\{\nu_m\} = \{0,1,0 \ldots 1,1\}$

$$E_{full} = E_c(n) + \sum_m \varepsilon_m \nu_m$$

$$P_{canon}(E,n) = P_{eq}(\{\nu_m\}|n) = \frac{1}{Z_n} e^{-\frac{E_{full}(\{\nu_m\})}{kT}} \qquad Z_n = e^{-\frac{F_n}{kT}}$$

- electrons tunnel independently

$$\Gamma_{full} = \sum_m \gamma_m^- \nu_m$$

L.P.Kouwenhoven, D.G.Austing, S.Tarucha “Few-electron quantum dots”

Small systems: Kinetic equation's coefficients

$$\frac{dP_n}{dt} = P_{n+1}W_{n+1}^{-} + P_{n-1}W_{n-1}^{+} - P_nW_n^{-} - P_nW_n^{+}$$

$$f(\varepsilon) = \frac{1}{e^{\frac{\varepsilon-\mu}{kT}} + 1}$$

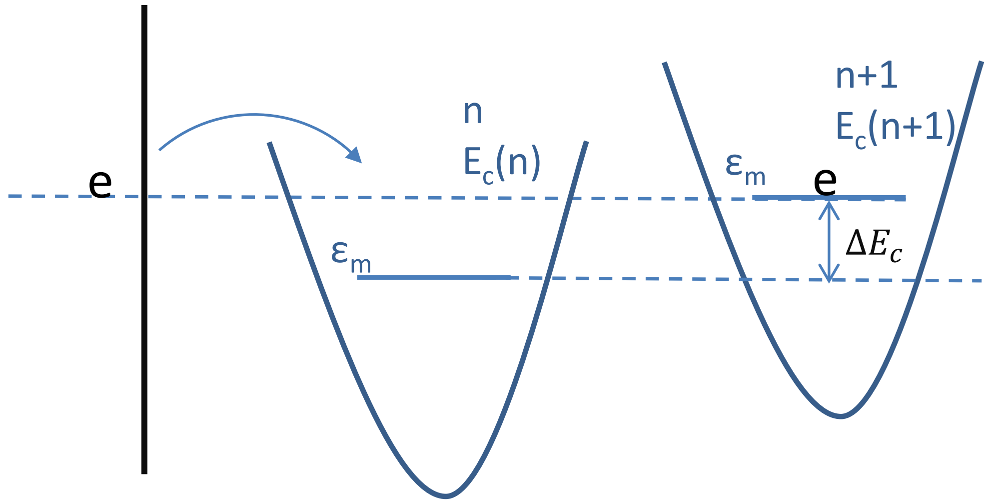
$$W_{n+1}^{-} = \sum_{\{\nu_m\}} P_{eq}(\{\nu_m\}|n+1) \sum_m \nu_m \gamma_m^{-}(n+1)[1 - f(\Delta E_c(n+1) + \varepsilon_m)]$$

$$W_n^{+} = \sum_{\{\nu_m\}} P_{eq}(\{\nu_m\}|n) \sum_m (1-\nu_m)\gamma_m^{+}(n) f(\Delta E_c(n+1) + \varepsilon_m)$$

$$W_{n-1}^{+} = \sum_{\{\nu_m\}} P_{eq}(\{\nu_m\}|n-1) \sum_m (1-\nu_m)\gamma_m^{+}(n-1) f(\Delta E_c(n) + \varepsilon_m)$$

$$W_n^{-} = \sum_{\{\nu_m\}} P_{eq}(\{\nu_m\}|n) \sum_m \nu_m \gamma_m^{-}(n)[1 - f(\Delta E_c(n) + \varepsilon_m)]$$

Small systems: ΔE_c



$$\Delta E_c(n+1) = E_c(n+1) - E_c(n)$$

$$E_c \propto n^2$$

$$\Delta E_c \propto n$$

Small systems: Microreversibility

$$\gamma_m^+(n) = \gamma_m^-(n+1)$$

$$W_{n+1}^- = \sum_{\{\nu_m\}} P_{eq}(\{\nu_m\}|n+1) \sum_m \nu_m \gamma_m^-(n+1)[1 - f(\Delta E_c(n+1) + \varepsilon_m)]$$

$$W_n^+ = \sum_{\{\nu_m\}} P_{eq}(\{\nu_m\}|n) \sum_m (1-\nu_m)\gamma_m^-(n+1) f(\Delta E_c(n+1) + \varepsilon_m)$$

$$W_{n-1}^+ = \sum_{\{\nu_m\}} P_{eq}(\{\nu_m\}|n-1) \sum_m (1-\nu_m)\gamma_m^-(n) f(\Delta E_c(n) + \varepsilon_m)$$

$$W_n^- = \sum_{\{\nu_m\}} P_{eq}(\{\nu_m\}|n) \sum_m \nu_m \gamma_m^-(n)[1 - f(\Delta E_c(n) + \varepsilon_m)]$$

Small systems: Solution

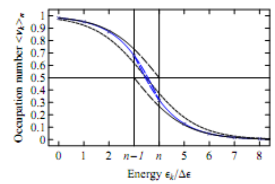


Figure 1. Comparison of different approximations for the canonical occupation numbers in a Fermi gas with equidistant spectrum, $n = 4$ and $\beta\Delta\epsilon = 1$. Crosses mark the exact values, thick (blue) continuous line shows the q-analog of Fermi-Dirac distribution, Eq. (19), thin continuous line — substitution $f_q \to f$ in Eq. (19), and short-dashed line — single Fermi-Dirac distribution, Eq. (3). The long-dashed lines between ϵ_{n-1} and ϵ_n are extrapolations of the corresponding functions into the gap $\mu_p < \epsilon < \mu_h$.

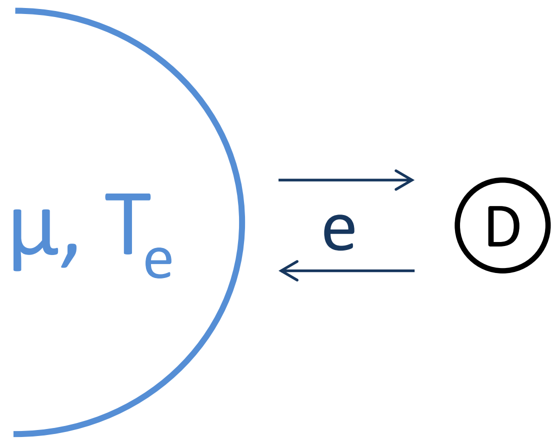
V. Kashcheyevs, "Exact canonical occupation numbers in a Fermi gas with finite level spacing and a q-analog of Fermi-Dirac distribution", arXiv:1110.6264 (2011)

Small systems: Solution

$$W_n^- = \frac{e^{\frac{\Delta E_c(n)-\mu}{kT}}}{Z_n}\left[\sum_{m=0}^{n-1} Z_m(-1)^{n-m-1}\sum_k \frac{\gamma_k^-(n)\, e^{-\frac{\varepsilon_k\,(n-m-1)}{kT}}}{e^{\frac{(\Delta E_c(n)+\varepsilon_k-\mu)}{kT}}+1}\right]$$

$$W_{n-1}^+ = \frac{1}{Z_{n-1}}\left[\sum_{m=0}^{n-1} Z_m(-1)^{n-m-1}\sum_k \frac{\gamma_k^-(n)\, e^{-\frac{\varepsilon_k\,(n-m-1)}{kT}}}{e^{\frac{(\Delta E_c(n)+\varepsilon_k-\mu)}{kT}}+1}\right]$$

Small systems: Solution check



Equilibrium

- microcanonical ensemble
- defined P_n and P(E)

$$0 = P_{n+1}^{eq} W_{n+1}^{-} + P_{n-1}^{eq} W_{n-1}^{+} - P_n^{eq} W_n^{-} - P_n^{eq} W_n^{+}$$

$$W_n^{-} = \frac{e^{\frac{\Delta E_c(n)-\mu}{kT}}}{Z_n}\left[\sum_{m=0}^{n-1} Z_m (-1)^{n-m-1} \sum_k \frac{\gamma_k^{-}(n)\, e^{-\frac{\varepsilon_k (n-m-1)}{kT}}}{e^{\frac{(\Delta E_c(n)+\varepsilon_k-\mu)}{kT}}+1}\right]$$

$$W_{n-1}^{+} = \frac{1}{Z_{n-1}}\left[\sum_{m=0}^{n-1} Z_m (-1)^{n-m-1} \sum_k \frac{\gamma_k^{-}(n)\, e^{-\frac{\varepsilon_k (n-m-1)}{kT}}}{e^{\frac{(\Delta E_c(n)+\varepsilon_k-\mu)}{kT}}+1}\right]$$

$$Z_n = e^{-\frac{F_n}{kT}}$$

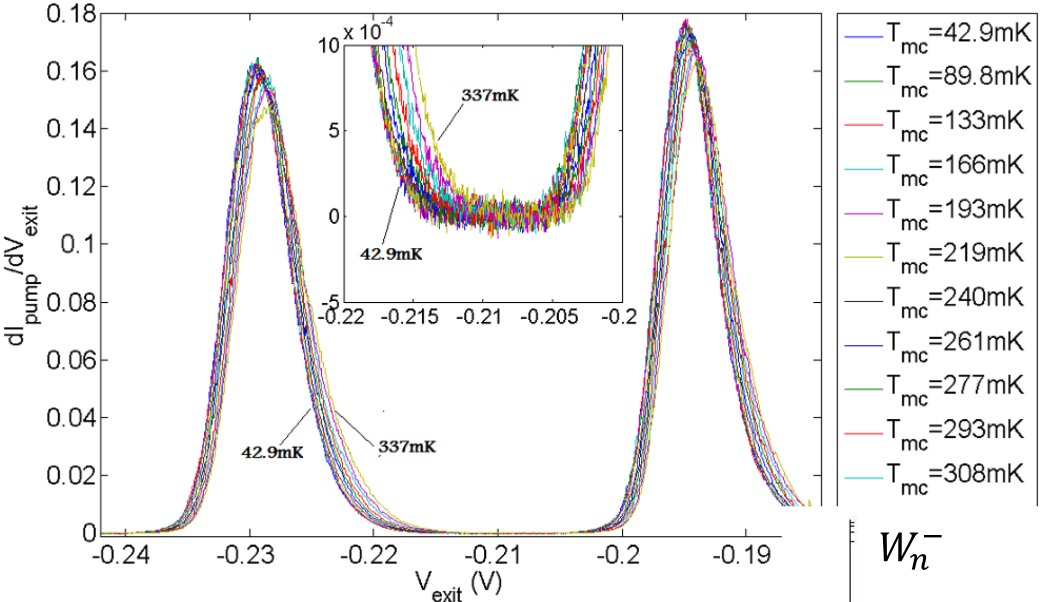
$$P_n^{eq} W_n^{-} = P_{n-1}^{eq} W_{n-1}^{+}$$

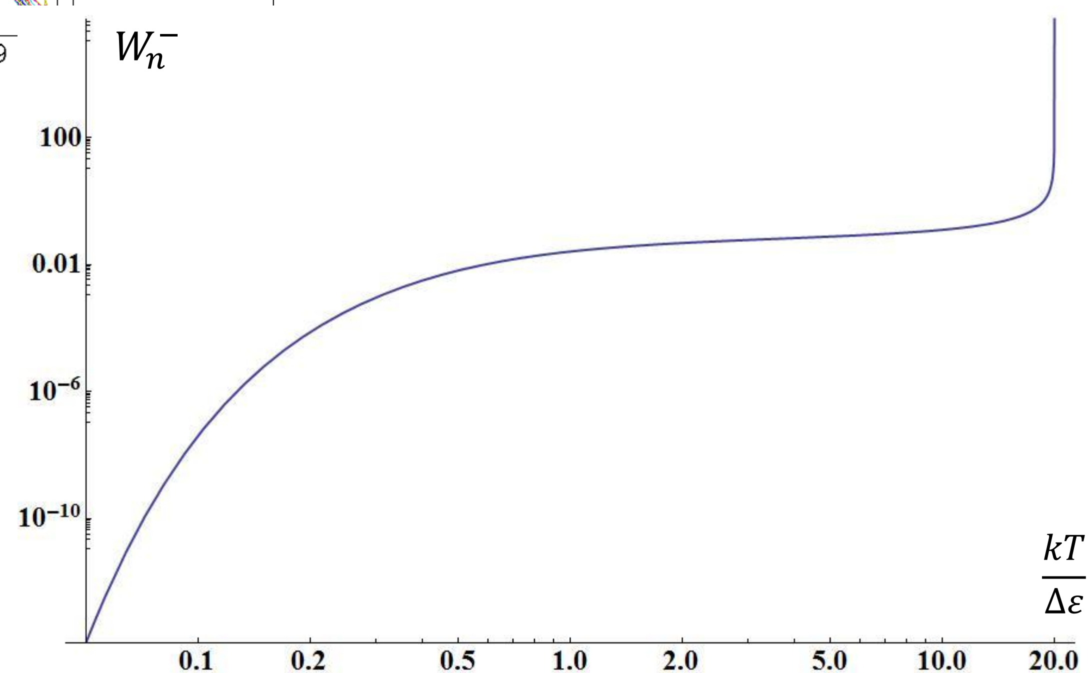
$$\frac{P_n^{eq\,-}}{P_{n-1}^{eq}} = \frac{W_{n-1}^{+}}{W_n^{-}} = \frac{1}{Z_{n-1}} \frac{Z_n}{e^{\frac{\Delta E_c(n)-\mu}{kT}}}$$

$$= \frac{e^{-\frac{F_n}{kT}-\frac{E_c(n)}{kT}+\frac{\mu\, n}{kT}}}{e^{-\frac{F_{n-1}}{kT}-\frac{E_c(n-1)}{kT}+\frac{\mu(n-1)}{kT}}}$$

microcanonical!

Small systems: Temperature in electron pumps

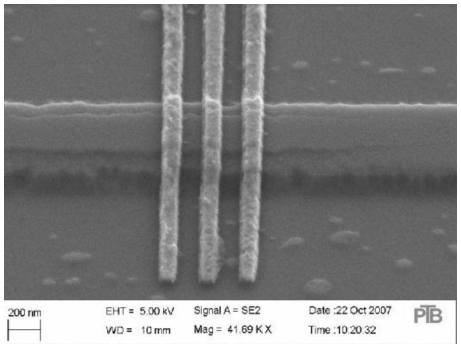


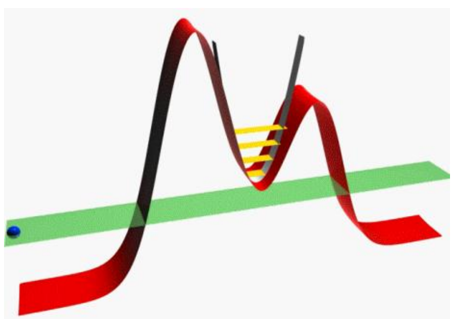


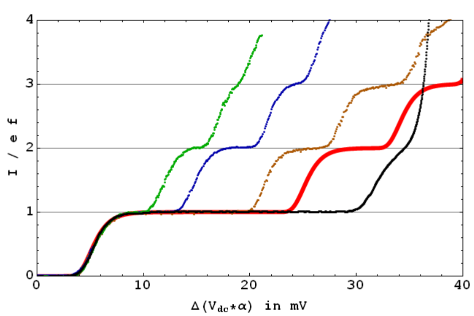
Data by M.Blumenthal (NPL), unpublished

Small systems: Examples

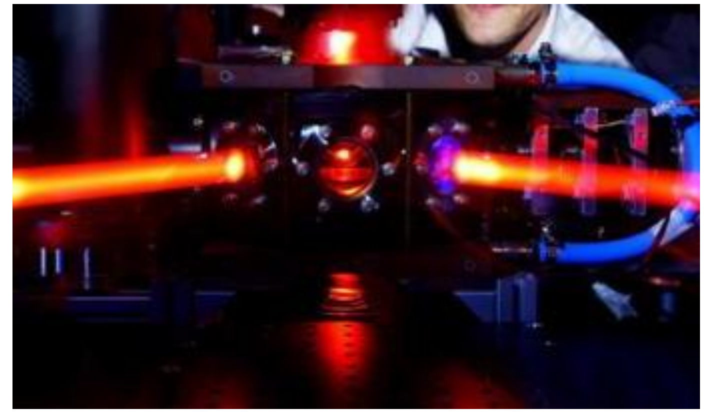
Electron pumps

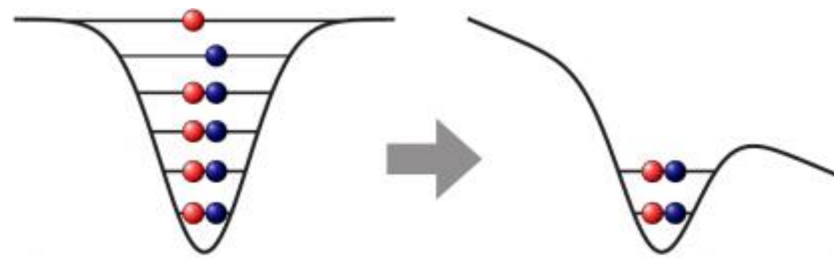


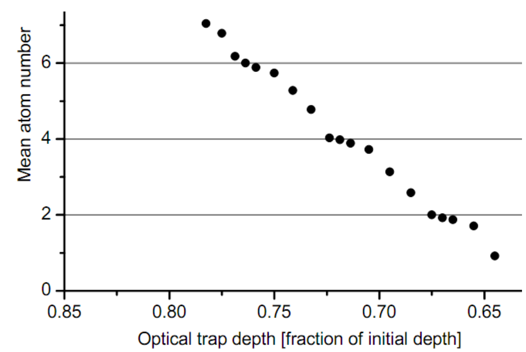




Ultracold fermion gases







L. Fricke , F. Hohls , P. Mirovsky , C. Leicht , B. Kaestner and others „Measurements of two sequential quantized-charge pumps“, Phys.Rev.B (2011)

F. Serwane, T. Lompe, T.B. Ottenstein and others “Deterministic Preparation of a Tunable Few-Fermion System”, Science (2011)